Helicopters and the Neutrality of Money

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Abstract. Models of monetary expansion, following Friedman (1969), tend to abstract away from relative price effects by assuming that the central bank distributes money directly to agents via helicopter. In light of the recent entertainment of helicopter drops as a potential monetary policy tool, this paper develops a computational heterogeneous-agent model to compare the relative-price effects of helicopter drops versus open market operations and avoid several limitations of standard cash-in-advance models. Helicopter drops are found to be significantly *more* distortive than open market operations. The results highlight the key role of financial systems in distributing changes in the money stock with minimal economic disturbance.

In 1969, Milton Friedman first used the famous "helicopter" metaphor for monetary policy. Granting that changes in the money stock are neutral with respect to relative prices in the long run, he argued, we can ignore relative price disturbances in the short-run too if we assume that new money balances are distributed directly to agents via a helicopter. Bernanke (2002) gave Friedman's simplifying assumption new life as a semi-tongue-in-cheek policy proposal for avoiding deflation at the zero lower bound. Since then, the idea has persisted on the inner fringes of macroeconomic discourse,[1] especially in the aftermath of nearly a decade of sluggish post-recession growth worldwide.

Since that time, the proposition that money is neutral in the long run but disturbing in the short run has been mostly papered over. Such effects may exist in principle, the conventional wisdom goes, but they are more or less quantitatively irrelevant, except perhaps in the case of a major inflation.

The conventional wisdom, of course, is conventional for a reason. Barring specific emergency measures, there has been little evidence of cash-induced relative price distortions causing any major inefficiency in a reasonably well-managed monetary regime. However, this fact

1 Among unconventional monetary policy proposals, it ranks perhaps less serious than forward guidance, but more serious than the trillion-dollar-coin.

is no reason to be sanguine on the prospects of helicopter money. Indeed, this paper argues that real life "helicopter drops" are likely to be significantly distortive. Up to this point, monetary expansions have been more or less neutral in practice precisely because they are *not* helicopter drops.

This paper sets up a stylized heterogeneous-agent model in order to highlight the relevant feature of Friedman's model that results in monetary neutrality: not the helicopter drop, but the *representative agent* construction. To relax this construction, we then formalize and implement an agent-based simulation with heterogeneity in the demand for cash balances, subject to random shocks either to the demand or the supply of money. To the extent that the distribution of demand for cash balances is not totally orthogonal to the distribution of demand for different types of goods, a helicopter drop will in fact be *less* neutral than traditional open-market operations, where newly created money flows into the economy through the banking system.

I. The State of the Literature

Changes in the money supply, whether stabilizing or not in practice, have – with few and mostly seigniorage-driven exceptions – always entered through or originated in the market of financial intermediaries.[2] Very little monetary expansion across the world consists in the literal printing of money. It is not insignificant that the supplies of national currencies everywhere in the world are managed by central *banks*.

The centrality of banking to stable currencies has been relatively underappreciated in economic theory. Indeed Friedman's helicopter thought experiment exists for the explicit purpose of ignoring the banking system, a tendency followed by many subsequent monetarists. On the other hand, those who do appreciate the significance of the banking system in originating and distributing changes in the money stock tend to lament the fact (e.g. Hayek [1933] 2008, ch. 4; see also Burns & Harwick [2016] for a survey of historical opposition to bank money).

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[2] Even under the classical gold standard, and despite the analytical predominance of the price-specie flow mechanism at the time, trade imbalances were predominantly offset in practice, not by price level changes leading to specie flow, but rather by internal changes in the quantity of bank money (Eichengreen 1995, p. 47).

Reinforcing this skepticism is a long line of models that feature real distortions resulting from monetary injections through open market operations (OMOs). In the background of these – usually implicitly, but sometimes explicitly[3] – is the assumption that a helicopter drop, whatever its practical demerits, would at least be less distortive.

Many of these models, unlike those with neutral helicopter drops, do feature agent heterogeneity. Fuerst (1992) divides agents into savers and borrowers, the latter of which benefit from changes in the interest rate induced by an OMO. Grossman & Weiss (1983) and Rotemberg (1984) divide agents into cohorts that withdraw cash from the bank in alternating periods, though Lucas (1990) shoehorns their model back into a representative agent construction because persistent wealth effects made them relatively intractable. An OMO in period t_1 then redistributes wealth to the agents who happen to be at the bank in that period.

Our model is similar in spirit, in that it divides agents into two classes between which the real effects manifest. The relevant heterogeneity is not spatial, however; rather, it consists in their demand for cash balances and for goods, a device that should simplify the model's interpretation and applicability. In addition, by using an agent-based simulation rather than an analytical cash-in-advance model, it sidesteps the tractability problems that result from persistent wealth effects.

In addition, our model has the attractive feature that the resulting real effects are not explicitly redistributionary (i.e. are not a direct transfer of wealth to one class of agents), a feature of earlier models that Fuerst (1992) called "a nuisance". In our model, the wealth distribution among agents is explicitly *not* changed. Instead, because the shocks we consider (a change in the cash demand of one cohort, and a transfer in proportion to existing wealth), the pattern of spending on the margin is changed without wealth effects. Instead, the primary effect is on relative prices. Though this may entail second order wealth effects in a more complete model, we abstract away from these by having all goods produced and sold by a single "store".

The model assumes a "spending" channel connecting monetary policy to real activity. Though investment channels have become analytically predominant in the past several decades of monetary policy analysis due to a weak link between measured monetary aggregates and real economic activity, there is increasing evidence that this failure is due to misaggregation of money

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[3] Grossman & Weiss (1983), for example, introduce their model by contrasting it against "a proportional transfer which would raise all nominal prices by the same percentage and thus have no real effects."

rather than to an intrinsically weak link (Hendrickson 2013; Barnett 2016; Harwick 2019). An investment channel entails a different set of relative price effects that will likely be invariant to distribution method. Though the model here does not explore the aggregation issue, the fact that spending shocks are transmitted in large part via assets generated in financial markets indicates the appropriateness of this more simplified model.

Finally, and most importantly, our model points to an important effect of monetary policy which has been overlooked to this point precisely because it is *not* a significant result of open market operations, but would be under alternative monetary distribution regimes. In this sense, by arguing for significant real effects of helicopter drops, it straddles the two literatures both on the real-effects of OMOs and the neutrality of helicopter drops.

In the following section we consider a stylized economy's dynamics in response to two different types of monetary shocks distributed via either a helicopter drops or an open market operation. Section III formalizes the argument from the preceding section, and section IV presents the results.

II. The Argument In Brief

Helicopter Drops

Imagine a small country unexpectedly winning a war. As fighting winds down, taxation relents, and confidence revives, the demand to hold the central bank's issues increases. In order to prevent downward pressure on prices, the central bank must expand its issues. After determining the exact magnitude of the adjustment necessary to stabilize nominal spending, it deploys a squadron of newly idle helicopters to disburse the money. By assumption, the increase in money supply is such in the aggregate as to exactly offset the increase in the demand for money. In the long run, therefore, no prices change. Our benchmark – monetary neutrality – is that no prices change in the *short run* either.[4]

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[4] To this point we have a single monetary asset. The banking model has segregated demand for assets, with consumers holding bank liabilities and banks holding reserves. This allows us to ignore real portfolio balance effects (Gurley & Shaw 1960), not because they are unimportant, but because they are extraneous to the real disturbances at issue.

For this to be true, it is necessary that the increase in the demand for money balances that triggered the expansion be *absolutely uniform* across all agents. Friedman's helicopter story, for example, is a representative agent economy (p. 5): Everyone has a uniform demand for money balances expressed in terms of weeks of income. Only in this case will spending patterns – and thus the structure of prices – remain undisturbed. Once he drops this assumption (p. 6), he is forced to concede "initial distribution effects" which preclude instantaneous and price-neutral equilibration.

To illustrate the significance of these effects, imagine our economy is composed of one half hobbits and one half dwarves, each group holding equal wealth in the aggregate. We will provisionally assume that helicopters are necessary because there is no banking sector, so there is no question of the composition of the demand for money, and we have a mint rather than a central bank. Hobbits, of course, are creatures of habit and essentially fixed in their spending habits, so following the defeat of Mordor, the revival of confidence causes only the dwarves to increase their demand for money.

Though the mint can calculate the precise *magnitude* of the required expansion, we assume it is constrained to a distribution proportional to agents' existing wealth.[5] The mint's helicopters therefore distribute half of the expansion to the dwarves themselves, and half to the hobbits. Because the dwarves constitute the entirety of the increased demand but only receive half the balances necessary to satisfy it, they continue trying to build up their money holdings, while the hobbits enjoy a windfall. In the short run, then, pickaxes continue to fall in price, and tobacco and raspberry jam begin to rise in price.

These price changes are disequilibrium phenomena. As the new money works its way through the economy, the dwarves will eventually succeed in building up their money holdings, and the hobbits will eventually spend off their windfall. As each group's spending returns to its previous pattern, the price of pickaxes must rise back to its pre-shock level, and the price of tobacco and raspberry jam must fall. In the meantime, however, the jam industry has been able to bid away resources from the axe industry, resources which must eventually be reincorporated at a cost into the production of axes.

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[5] Other patterns of distribution – for example uniformly dividing the sum among agents regardless of their current balances – do not affect the basic result. See Section VI below.

This brief story illustrates our general principle: when the distribution of the monetary expansion does not match the distribution of the change in demand for money balances, some people whose demand has not increased will receive a windfall, while some whose demand has increased will remain unsatisfied. This holds whether the increase is distributed proportionally, as a lump sum, or *in toto* to some favored group. Whether or not the overall price index is disturbed, the initial changes must eventually reverse themselves as equilibrium is restored. Even if the mint knows the precise quantity necessary to compensate for changes in demand, unless it also knows the *pattern* of those changes, it cannot avoid the misdirection of resources associated with money-induced relative price changes.[6] Knowing the precise magnitude of the necessary adjustment is therefore a necessary condition, but *not* a sufficient condition, of monetary neutrality in the face of such changes.

Of course, even knowing the magnitude is a heroic assumption. In practice many monetary expansions will be better described not by an equilibrating monetary response to a demand shock, but to an exogenous and *disequilibrating* monetary shock, the situation analyzed by Friedman (1969, sect. III) in his original use of the helicopter metaphor. This modification does not essentially change the story. The main difference is that, at the end of the process, the overall level of prices must be higher. However, we may generalize the principle from the equilibrating situation: whether or not anyone's demand for money balances changes, a relative-price neutral monetary expansion must be distributed in proportion to each agent's *disposable* cash, i.e. the balances they hold at any point in time in excess of their desired balances. A distribution in proportion to something else – for example total wealth or existing money

[6] This echoes the discussion in Mises (1966: 412): "We may, if we like, assume that every member gets a share of the additional money right at the moment of its inflow into the system . . . The result will be that changes . . . never affect the prices of the various commodities and services to the same extent and at the same date." He also considers countervailing changes like those we are considering now, that leave purchasing power unchanged but nevertheless disarrange prices, at least temporarily (his example is the productivity of gold mining increasing in lockstep with money demand). Later (p. 793) he discusses credit expansion, somewhat critically. Unfortunately he did not explicitly compare the two mechanisms.

Simmel ([1907] 1978: 161) also had an intuitive grasp of the logic here: "real success [in a neutral monetary expansion] can be achieved only by an unequal distribution of the increased supply of money, at least in the first instance."

holdings – will affect the amount of disposable income *disproportionately*.

Suppose, therefore, that hobbits' demand for money holdings is twice that of dwarves at the current price level – say, $1,000 and $500 respectively. Each agent has a net worth of $1,500. Each hobbit therefore has $500 in disposable cash and each dwarf $1,000. The central bank then disburses $1,000 to each agent. Each dwarf's disposable cash has doubled, but the hobbits' disposable income has *tripled*.[7] The pattern of spending, to the extent that it is not totally orthogonal to the patterns of money demand, is affected by a helicopter drop in such a way that it must in turn affect relative prices and resource allocation.

Open Market Operations

One of the most important functions of a banking sector is to obviate the need for the monetary authority to know the exact pattern of disposable incomes. However, introducing a banking sector complicates the analysis somewhat, as the demand for money may be met by a number of assets varying in liquidity, and exchangeable one for another at a fixed price. Let us first of all limit our discussion to two generic types of money assets: base money and bank liabilities, the latter being issued on a fractional reserve basis as a promise to pay the former, and (therefore) the former being more liquid. For concreteness, I will refer to cash balances and deposit balances, respectively.

In addition, we must distinguish between the transactions demand for money, for which cash and deposits are approximately perfect substitutes, and liquidity demand, for which they are *not* substitutes. In a well-functioning banking regime, an increase in transactions demand is typically accommodated by an expansion in bank money (Selgin 1988, ch. 5). A lower gross volume of clearings allows the bank to safely expand loans and issues up to the point that nominal spending is stabilized (Selgin 1994). If banks are not prevented from doing so, there is no need for central bank action at all, and no relative prices need be disturbed. We therefore leave aside the case of an increase in transactions demand.

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[7] Alternatively, in flow terms, suppose each agent has an equal income and spends all its disposable income each period. In equilibrium, therefore, hobbits are wealthier than dwarves. Accordingly, they receive more from a proportionate transfer. Because the whole of the transfer is reckoned disposable income before prices rise, this must be counted a windfall for the hobbits relative to dwarves.

An increase in the demand for liquidity is a more difficult case. In our hypothetical banked economy, banks issue liabilities redeemable for cash on a fractional-reserve basis. An increase in the demand for cash balances will contract the banks' balance sheets, making them less liquid and (assuming no excess holdings of prudential reserves) forcing them to contract loans, effecting a contraction in deposit balances *larger* than the increased demand for cash balances, a problem variously referred to as multiple contraction or the perverse elasticity of credit.[8]

Liquidity runs are for this reason among the more disastrous events that can befall a modern financial system. If the run cannot be averted, the only market mechanism for returning to monetary equilibrium is a deflationary recession.

Most discussion of the problem has focused either on central bank responses to a liquidity run, or on prophylactic measures. So keeping (for now) the assumption that the central bank can respond with an injection of liquidity of the appropriate size, let us compare the central bank's response to an increase in the demand for cash balances with its response in the helicopter scenario.

Suppose, then, that following a liquidity run the Bank of Gondor ignores the idle helicopters and instead expands the money supply the traditional way by buying assets from banks on the open market. Any long-run windfall accrues to the originator of the asset as seigniorage – to the government in the case of treasury bills, or to the bank in the case of bank liabilities – which is able to issue debt at a lower interest rate. The financial system as a whole has not necessarily received a windfall, but it *has* become more liquid – i.e. it is able to more readily exchange (illiquid) interest-bearing assets which its customers will not accept for non-interest-bearing assets (cash) which its customers will accept, without incurring "fire-sale" losses.

This "shoring up" obviates the necessity of contracting the bank's issues in the event of a liquidity run: if the central bank is able to inject just enough liquidity into the banks to offset the increase in the demand for cash balances, no changes in the size of banks' portfolios, in the

[8] This problem turns out to be even more significant the more monetary assets we take under consideration, as there exist assets leveraged on top of bank deposits and other financial assets that will be forced to contract in the event of a liquidity run. See Harwick (2018).

pattern of consumer spending, or (therefore) in relative prices, need occur.[9] Those whose demand has increased withdraw cash from their banks, and these banks in turn will bid more for liquidity from the central bank. The banks which would have been forced to restrict their lending by raising their interest rates now find it unnecessary to do so. The central bank, for its part, is relieved of the necessity of matching the *pattern* of the change in demand: that task is taken care of by the banks themselves.[10]

Likewise, even when the open market operation is disequilibrating, banks can intermediate additional liquidity to those whose demand at the margin is highest, i.e. in proportion to *disposable* cash flows. Thus we will still expect to see less purely monetary disarrangement of relative prices based on idiosyncratic spending habits. Prices must rise more uniformly when inflation is carried on through financial markets than by helicopter drop.[11]

A banking system, to say nothing of a full-fledged financial market, introduces a number of considerations that a more complete model would account for, in particular the changes in investment induced by changes in interest rates that have been the focus of much recent work on monetary policy transmission. Our present aim, however, is not to evaluate the significance of all the various channels through which monetary expansion runs, but − more modestly − to highlight a neglected channel that tips the balance of evaluation on the margin away from helicopter drops and toward traditional open market operations. We therefore leave investment

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[9] The seigniorage may disarrange relative prices as it is spent off, but this quantity is small compared to the expansion effected (or the contraction avoided) in the broader money supply, so its effect should be relatively minor.

[10] It should be noted that the existence of a banking sector does not affect the conclusions of the previous section if the central bank decides to use helicopters after all. If the dwarves initially register their demand for cash balances by withdrawing deposits from their banks, the banks will again be forced to contract their loan portfolios. In this case, the demand is simply transformed into a demand for liquidity on the part of the banks. The new money still constitutes a one-time redistribution, and must still find its way circuitously back to the banks, effecting price changes as it goes.

[11] It is worth distinguishing potentially complementary explanations for price stickiness. In one account, the signal to raise prices reaches some merchants sooner than others. In another, the signal reaches everyone at once, but some merchants still fail to respond with the appropriate price change for whatever reason. A banking system mitigates the former issue, but *not* the latter.

to the side in order to focus on changes in spending.

III. The Model

The relative price neutrality of helicopter drops in Friedman's original model depends on the homogeneity of agents' demands for money balances – i.e. it depends on a representative agent style model. By contrast, we use a computational agent-based simulation to introduce heterogeneity in order to focus on the dynamics of equilibration rather than comparative statics, and (in the banking model) to sidestep the tractability problems that have hindered earlier attempts at modeling the distributional effects of open market operations. If the helicopter drop results in relative price effects even in such a simple and stylized model, the relevance of its traditional pedagogical use – abstracting away from the distribution and relative price effects of monetary expansion in the context of a representative agent model – is severely circumscribed.

The model follows the logic in the preceding sections closely. This section adds formal detail about the operation of the model, and the following section presents quantitative results from the simulation.[12]

The Helicopter Model

We begin with a set of N agents indexed by i and defined by the state vector $A_i = \{g_i\ ,\ b_{t,i}P_{t,g}\}$, where $g_i \in \{j, a\}$ is the agent's type, and $b_{t,i}P_{t,g}$ is the agent's nominal cash balances at time t.

Each period opens with the store producing goods and adding them to an inventory $V_{t,g}$. The store then sets its prices $P_{t,g} \in \{P_{t,j}\ ,\ P_{t,a}\}$ in order to hold enough inventory to meet typical demand on the one hand, and avoid holding unnecessary inventory on the other hand. $P_{t,g}$ is the vector of the store's choice variables with which it aims for a target inventory $V^*_{t\text{-}\tau,g}$, which is one standard deviation above the mean of total demand $D_{t,g} \equiv \sum_{g_\gamma = g} d_{t,\gamma}$ over the last τ periods.

(1) $$P_{t+1,g} = \max\{a(P_{t,g}, v_{t,g})\ r(\Delta V_{t,g})\ , 0\}$$

(2) $$v_{t,g} \equiv V^*_{t\text{-}\tau,g} - V_{t,g} + s_{t,g}$$

where $v_{t,g}$ is the inventory shortfall of good g at time t. The function a (for 'absolute') is defined

[12] The latest version of the model can be downloaded at www.cameronharwick.com.

such that $\partial a/\partial v > 0$ and $a(P_{t,g}, 0) = P_{t,g}$: excess inventory (i.e. $v_{t,g} < 0$) results in a falling price, and short inventory (i.e. $v_{t,g} > 0$) results in a rising price, with any shortage $s_{t,g}$ being counted toward the difference as well. The function r (for 'relative') is linearly increasing in $\Delta V_{t,g}$ and defined such that $r(0) = 1$. This adjusts the price based on the magnitude of the last period's change in inventory (thus $\Delta P_{t,g}$ is decreasing in $\Delta V_{t,g}$). $P_{t+1,g}$ is bounded from below by zero. This structure implicitly embodies stabilizing price expectations and avoids systematically overshooting the equilibrium price.

The store also employs all agents in the production of jam and axes. Agents each supply one unit of labor per period. Production is a linear function of labor supply. The store apportions its labor each period between jam and axe production in the proportions $\rho_{t,j} \equiv \rho_t$ and $\rho_{t,a} \equiv (1-\rho_t)$ as a decaying average of the target capital proportion $\rho^*_{t,g}$. The rate of decay is governed by a factor immobility parameter $k \in \mathbb{R}^+$, such that capital is perfectly mobile (i.e. $\rho_{t,g} = \rho^*_{t,g}$) at k=0 and perfectly immobile at k=∞.

(3) $$\rho^*_{t+1,g} = \frac{P_{t,g}}{P_{t,a} + P_{t,j}}$$

(4) $$\rho_{t+1,g} = \frac{k\rho_{t,g} + \rho^*_{t+1,g}}{k + 1}, \quad \rho_0 = 0.5$$

Each good's production is multiplied by a productivity parameter μ_g and added to the store's inventory.

(5) $$\Delta V_{t,g} = \mu_g \rho_{t,g} N - D_{t,g}$$

Thus, the portion of labor devoted to the higher-priced good increases with the price differential. This indicates capital being bid away from the production of one good into production of the other, and serves in the long run to dampen price differentials compared to the case where the ratio of productive effort devoted to each good is fixed.

The store calculates its target cash reserves $b^*_{t,s}$ as one period of cash flow, specifically, the cash necessary to pay one period's wages (6). Target wages w^*_t, in turn, are calculated as the store's total excess balances divided by the labor force (7).

(6) $b^*_{t,s} = w_{t\text{-}1,i}N$

(7) $w^*_t = (b_{t,s} - b^*_{t,s})/N$

The actual wage w_t is calculated as a decaying average of target wages as in (4), the rate of decay being governed by a wage stickiness parameter $m \in \mathbb{R}^+$. Agents are subjected to wage shocks each period, receiving a random amount $w_{t,i}$ normally distributed around w_t with a standard deviation of $0.5w_t$.

The agent population $N \in \mathbb{N} > 2$ is split evenly between agents who consume the goods in g. All agents maximize a constant elasticity of substitution utility function in consumption of the desired good, g, and real balances, b.[13]

$$(8) \qquad u_{t,i}(b,d) = \left(\beta_g^{\frac{1}{\sigma}} b_{t,i}^{\frac{\sigma-1}{\sigma}} + (1-\beta_g)^{\frac{1}{\sigma}} d_{t,i}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where $b_{t,i}$ is agent i's real balances at the end of period t, $d_{t,i}$ is the his consumption of the desired good that period, σ is the elasticity of substitution between goods and real balances, and β_g is the weight each agent-type places on real balances as opposed to consumption. The agent's budget constraint in real terms is

(9) $b_{t-1,i} + w_{t,i}/P_{t,g} = b_{t,i} + d_{t,i}$

meaning that the agent's real balances at the start of the period can be kept or spent on consumption. Because expected utility is based on previous utility and is not affected by current-period decisions, if we define $b^*_{t,i}$ as the left-hand side of (9) (i.e. the agent's balances at the beginning of t) maximizing (8) subject to (9) causes the last term of the former to drop out and gives us the simple equimarginal conditions,

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[13] There are a number of special cases of the CES utility function. At σ=0 the two inputs are perfect complements and the function becomes a Leontief function $u_{t,i} = \min\{ b_{t,i} , d_{t,i} \}$. At σ=∞ the two are perfect substitutes and the function becomes linear. At σ=1 the utility function becomes a Cobb-Douglass function $u_{t,i} = b_{t,i}^{\beta} d_{t,i}^{(1-\beta)}$. Intuitively, real balances and consumption are not particularly good substitutes, so the simulations in the following section use a low value of σ=0.5. However, because σ drops out in the first-order conditions, u is the only variable affected by the value of σ, and therefore the specific value matters only for a welfare analysis and not for the system's dynamics.

(10) $d_{t,i} = (1 - \beta_g)\, b^*_{t,i}$

(11) $b_{t,i} = \beta_g\, b^*_{t,i}$

For a given set of productivity parameters, β will correspond to a unique target real balance $\bar{b}_g$ (see the appendix for a proof) toward which actual money holdings $b_{t,i}$ will tend. As $\bar{b}_g$ is proportional to $\beta/(1-\beta)$, with the former approaching infinity as β approaches 1, in order to maintain comparability with linear shocks to money balances, we shock $\bar{b}_g$ to affect the demand for money, leaving β_g endogenous.

Agents purchase $d_{t,i}$ from the store sequentially in a random order, drawing down the store's inventory as they go. If the store runs out of inventory, the current agent purchases the remaining inventory, and its excess demand along with subsequent agents' demand for the good remain unsatisfied that period and are added to $s_{t,g}$.

Finally, there exists a central bank, which has the option of targeting a level either of $M \equiv b_{t,s} + \sum(b_{t,i})$ or NDGP=$\sum(D_{t,g}P_{t,g})$, the other being endogenous. Any changes in the money supply are disbursed directly to agents via helicopter in proportion to their existing holdings:[14]

(12) $\Delta b_{t,i} = \Delta M\, b_{t,i}/M$

The Banking Model

The model with banking is substantially similar, except that agents hold bank liabilities instead of cash, and have the option to make consumption-smoothing intertemporal transactions, i.e. to take out loans. The store withdraws its balances at the end of each period and pays the agents in cash, who then deposit their cash at the beginning of each period.

Agents calculate at the beginning of each period their expected consumption as a decaying average of the quantity purchasable with their disposable income. If the consumption they can afford based on this period's wages is below expected consumption, they can take out a loan from the bank. Consumption is smoothed according to a special case of the formula in Hall (1978), with expected future consumption $E_{t,i}[d]$ equal to the agent's mean realized wage over

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[14] This method is in order to simulate the distribution method most often supposed to be neutral. The results do not change if the central bank divides the sum equally among agents, though this also entails a more explicit redistribution.

the last τ periods divided by $P_{t,g}$.

$$(13) \qquad d_{t,i} = \frac{r}{1+r}(1-\beta_g)b^*_{t,i} + \frac{1}{1+r}\,\mathrm{E}_{t,i}[d]$$

where r is the real interest rate. The function is defined so that the agent consumes his expectation at r=0, and his current disposable income at r=∞. The difference between his expenditure and his disposable income is borrowed if positive, and repaid if negative and if the agent has a positive outstanding balance $L_{t,i}$. The change in borrowing each period is therefore defined as

$$(14) \qquad \Delta L_{t,i} = (d_{t,i} - \mathrm{E}_t[d])\, P_{t,g}$$

Combining (13) and (14) gives us the individual agent's demand function for credit each period,

$$(15) \qquad \Delta L_{t,i} = \frac{r}{1+r}\left((1-\beta_g)b^*_{t,i} - E_t[d]\right)P_{t,g}$$

If $d_{t,i} > \mathrm{E}_t[d]$, the agent amortizes his outstanding debt by the difference, in which case $\Delta L_{t,i} < 0$. The bank requires the interest on any outstanding loans to be paid each period, though agents can default if their balances are insufficient to cover the difference, in which case their outstanding balance is halved.

Finally, the bank adjusts its interest rate i using a formula analogous to the store's price equation in order to hit its target reserve ratio, calculated as two standard deviations above mean withdrawals over the last τ periods. Because all goods transactions go through the store, the reserve ratio tracks the mean and variance of aggregate demand. Interest is assessed on loans each period, and profits are disbursed back to agents in the form of interest on their deposits. The real interest rate is bounded from below by zero.

IV. Results

Each model is subjected to reequilibrating and disequilibrating shocks in turn. For the reequilibrating shocks, dwarf money demand $\bar{b}_g$ is shocked with 2% probability each period, with a standard deviation of 4 centered around the current value and bounded from below by 1. The central bank then observes nominal spending $Y=\sum_g D_g P_g$ each period and responds to deviations from a constant target by deploying helicopters (or heli-vacuums, as necessary) or open market operations to bring nominal spending back to target.[15] The helicopter drop, therefore, *re-equilibrates* real output and avoids the longer process of equilibration through the price level.

The disequilibrating shocks, on the other hand, are an unconditional shock to the supply of base money. With 2% probability each period a random amount of money with a mean of 0.01M and a standard deviation of 0.15M is distributed to (or siphoned away from) agents either through helicopters or an open market operation. Though the mechanism was more clearly visible in the exposition of the equilibrating adjustments, the results of the disequilibrating adjustments are more straightforward to interpret. We therefore present the results in reverse order from the exposition of the mechanism.

For both runs, we initialize the model with constant benchmark parameter values[16] and run it for 10,000 periods. For the disequilibrating shock, the impulse response functions are constructed from the effect of a shock to $\log(M)$ on $\log(P_j/P_a)$.

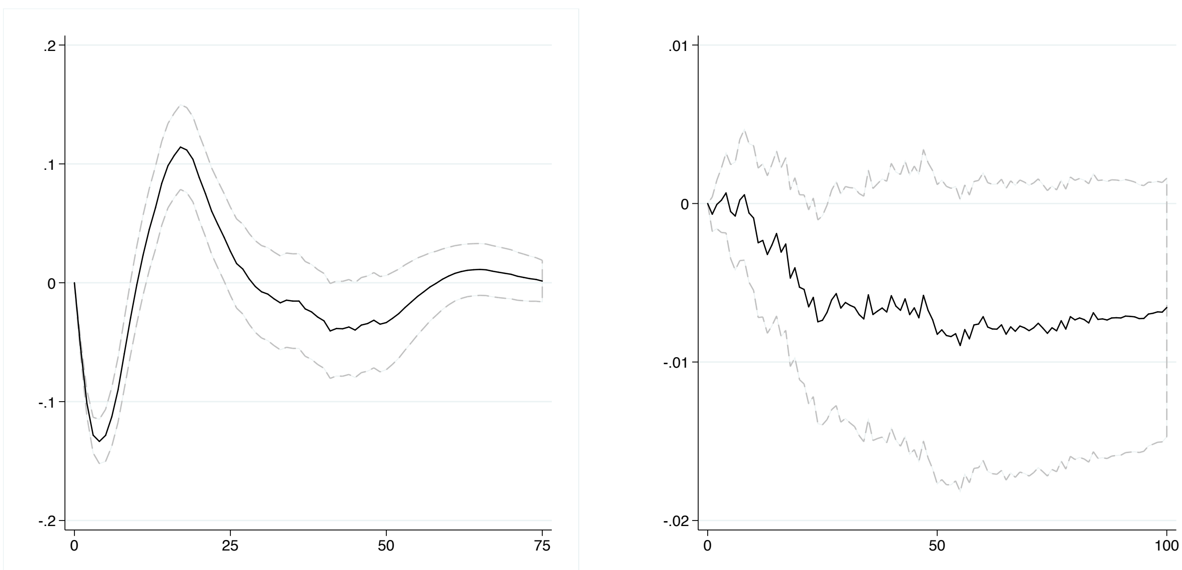

Fig. 1: impulse response functions for the log of Jam/Axe relative price following a shock to base money under disequilibrating helicopter drops (left) and an open market operations (right). 95% confidence intervals are indicated by dashed lines.

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[15] In the absence of productivity shocks, a nominal spending target is functionally equivalent to a price level target.

[16] $N = 100$, $\mu_j = \mu_a = 1.75$, $\sigma = 0.5$, $k = 100$, $\beta_a = 0.8$, $\beta_j = 0.2$, and $b_0 = 1{,}000$. Other values do not change the basic results.

The effect of a monetary expansion on relative prices is far more pronounced under a helicopter drop than under an open market operation. Though both affect the price in the same direction, the effect is an order of magnitude smaller (note the scale on the vertical axis) and statistically insignificant when the money is injected through the banking system rather than directly to agents.

For the reequilibrating shocks we control for dwarf cash demand in order to isolate the effect of the money supply shock, though because the expansion is so strongly correlated with the cash demand shock the effect cannot be strictly isolated. For this reason the helicopter drop actually disturbs relative prices *more* severely and for a longer period, despite moving the economy as a whole toward monetary equilibrium: the helicopter drop compounds the disturbance caused by the change in cash balances.[17] The open market operation, again, shows an effect an order of magnitude smaller, and insignificant after 20 periods – though there is a small effect at the beginning due to the fact that nominal spending must fall in order to trigger monetary expansion at all.

[17] The price increase for jam following an increase in dwarf cash balances actually depends on sufficiently high capital immobility and wage stickiness. With perfectly mobile capital, the price decrease of axes directs production towards jam, dampening the fall in axe price and depressing jam price as well. With perfectly flexible wages, the drop in spending is reflected fully and immediately in wages, which ends up depressing both prices. While I assume these values based on the plausibility of some stickiness and immobility, the significance of the result does not depend on either. Even if both prices fall, for either reason, the price of axes falls more – i.e. we still observe a drop in the *relative* price of axes.

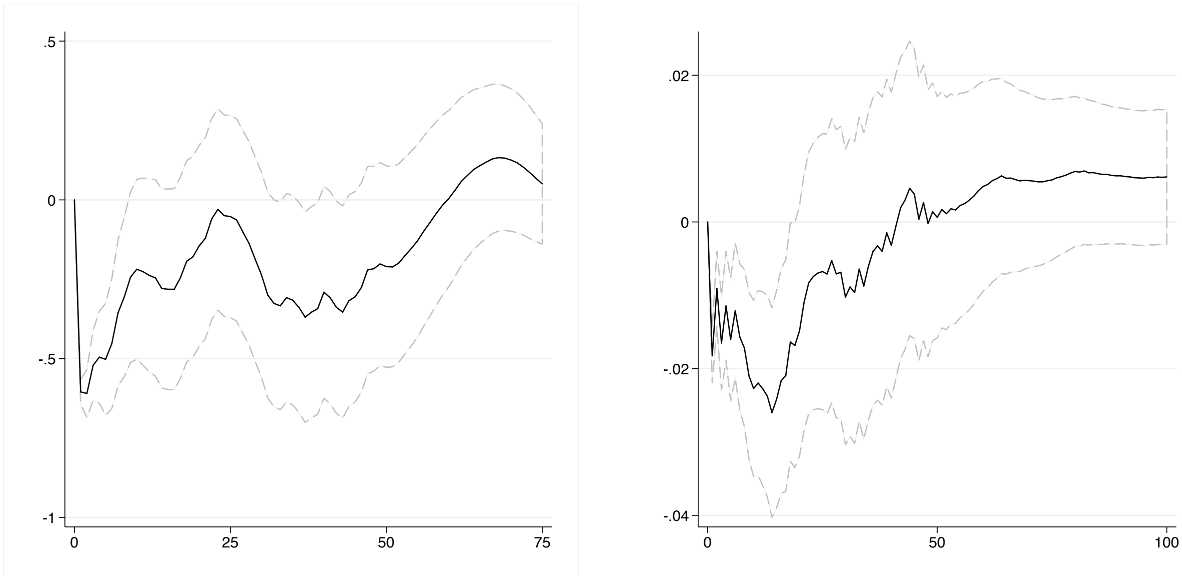

Fig. 2: impulse response functions for the log of Jam/Axe relative price following a shock to base money under reequilibrating helicopter drops (left) and an open market operations (right) in response to a shock to dwarf cash demand.

The responses of the model to money supply shocks on other variables such as the price level, capital allocation, utility, and debt load (in the banking model) are all in the expected direction and are available upon request. One final point bears mention on the welfare implications of the model: because capital allocation is a function of a decaying average of the relative price, a disturbance to relative prices will misdirect capital, depending on the value of the capital immobility parameter.[18] It is important to note that a response to a temporary change in relative prices is economically wasteful relative to a situation where relative prices were not disturbed, *even if that response is optimal* given the change in relative prices.

This is precisely the problem that a banking system solves. If a demand for cash balances can be registered to private banks who are equipped and incentivized to accommodate that demand and act as a cushion for spending, no relative price disturbances need occur, the volume of spending need not fall, and no capital need move from the use indicated by the pattern of consumer demand.

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[18] Because the capital immobility parameter reflects the cost of reallocating capital from one use to another, it should not be assumed that a higher degree of capital immobility means less waste.

V. Discussion

The Costs of Fiscal Policy

To the extent that the helicopter drop represents a policy proposal rather than a simplifying assumption, it is usually used as a metaphor either for the monetization of fiscal policy or for the transfer of newly created base money directly to consumers. From the standpoint of this paper, both proposals are similar in the sense of increasing spending by giving some group – a subset of consumers, or the government – a windfall to be spent off, in a way that fails to match the pattern of money demand on the margin.

The idea has an intuitive appeal. If the problem is lax consumer spending, the banking system seems like an unnecessary interposition. Especially if banks are reluctant to lend, why not skip the middleman entirely and give the money directly to the people we'd like to spend it?

There exist a great number of arguments for and against fiscal stimulus and direct-to-consumer monetary stimulus, involving both political economy and welfare economics considerations. Economic inefficiency arising from relative price distortions are unlikely to be the decisive consideration in any particular helicopter drop proposal. However, as a heretofore underappreciated cost, it should discourage such programs at least on the margin, if not on the whole.

Cryptocurrencies

In addition to the use of so-called helicopters as an anti-deflationary tool, the comparison developed in this paper also has direct relevance to the recent development of cryptocurrencies, which provide something like a small, open, and unbanked economy, in which money supply adjustments must be distributed by a fixed rule rather than through a banking system.[19]

To date, and despite a great deal of innovation *around* the problem, there are no circulating assets leveraged on top of a cryptocurrency base. There is thus no way to accommodate shifts in currency demand except by a change in the value of the currency – hence a tremendously volatile exchange rate. The daily change in the BTC/USD exchange rate has reached nearly 50% in both directions on multiple occasions – the EUR/USD rate, by contrast,

[19] This section draws on and expands the argument in Harwick (2016).

never changed by more than 2% in a day over the same period. For this reason, exchange rate stability by management of the money supply has become something of a holy grail for cryptocurrency protocol design.

Because the protocol constituting the currency is also the means of transaction, the velocity of money is available as a parameter to the protocol, and the money supply can respond to shifts in money demand automatically and with a precision unavailable to central banks. Our apparently unrealistic assumption in Section I that the total quantity of helicopter money necessary to compensate a fall in spending could be precisely determined could, therefore, in principle, be overcome programmatically by a well-designed cryptocurrency. Other creative solutions have also been offered to fix the exchange rate to (for example) the Dollar.

Without a banking system, a cryptocurrency protocol must specify two monetary rules: 1) a rule by which the quantity of money is determined, and 2) a rule by which changes made necessary by the first rule are distributed. Both types have been discussed, though the macroeconomic effects of the latter have so far been much neglected; the case for one rule over another usually hinges on technical or fairness considerations. Possible distributive rules include the following, any of which can in principle be joined to a quantity rule that stabilizes the flow of nominal spending:

- Distributing new coins *in toto* to miners as a reward for verifying transactions. This is a natural extension of the Bitcoin protocol. Decreases in the money stock must in this case be borne elsewhere, so mandatory transaction fees which are then destroyed (as in Peercoin) have been proposed.

- A variant of this, based on the proof-of-stake paradigm, is to distribute coins in proportion to the product of coin balances and time on the network, again as a reward for verifying transactions (King & Nadal 2012).

- A protocol-level democratic mechanism by which users vote on the direction of new funds (Lee 2014), analogous to the use of fiscal policy for macroeconomic stabilization.

- Finally, there is "rebasing" (Ametrano 2014), in which nominal coin balances are scaled up and down automatically in order to stabilize the purchasing power in terms of some other currency or a basket of goods. This is the closest practical analogue to a "pure" helicopter drop.

It follows from the foregoing analysis that none of these norms can in fact mitigate relative price distortions arising from changes in the monetary base, *even assuming the necessary magnitude of that change can be perfectly calculated*.[20] Without a banking system, the automatic behavior of the money stock comes only at the expense of the automatic distribution of liquidity to those who demand it. To introduce elasticity to the supply of cryptocurrency will not necessarily reduce monetary disequilibrium, purchasing power volatility, or exchange rate volatility from the high levels thus far observed in Bitcoin and other cryptocurrencies.[21]

Implications for Monetary Regimes

The analysis of this paper also suggests sensible reforms for central banks. In the first place, while an increase in the demand for cash balances is equivalent to an increase in the demand for liquidity in the current monetary regime, this is not necessarily true across monetary regimes. While cash is the only base money readily usable for transactions by the public, deposits may be a poor substitute for cash in some cases for reasons other than the latter's higher liquidity. There are, in addition, important physical characteristics such as anonymity and physical transferability.

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[20] It will not do to apportion the changes among finer classes according to their pattern of spending, either. The protocol must then know *ex ante* how many groups to divide the users into, and must have a rule placing each user in a definite group. Under free and anonymous registration, it is difficult to imagine the ecosystem avoiding collapse under the weight of rent-seeking for new funds. In fact, if rational expectations means agents have the "true" model of the economy in mind when forming expectations, an open-source codebase will imply rational expectations in a more or less literal sense, which in turn implies that such a protocol could never get off the ground in the first place.

[21] One caveat on the application to cryptocurrency bears mention. The relative price and capital allocation effects will not be evident in a currency that does not serve as a unit of account. Most Bitcoin transactions, for example, are priced in Dollars and converted to Bitcoin at the point of sale. Because prices are perfectly flexible, monetary disequilibrium comes through fully and immediately in the exchange rate. In addition, because relative prices are pinned down by a far larger volume of Dollar-denominated activity, no mispricing or capital misallocation need occur. In other words, cryptocurrencies can avoid the most severe consequences of monetary disequilibrium *only so long as they remain niche currencies.* As soon as they begin to take on unit of account functions, the stakes of monetary disequilibrium become much higher, and real economic distortions must result.

Nevertheless, if more liquid monies are the only way to satisfy the demand for such characteristics, the total money supply will be subject to needless shocks.

> A rise in [the demand for cash balances] is a routine occurrence which does not involve any loss of confidence in banks; it can in theory be satisfied by a circulating form of [bank liabilities]. In contrast, a rise in [base] money demand means a demand to exchange [bank liabilities] for [base] money, the ultimate money of redemption. In a closed system this implies either a loss of confidence in banks issuing [liabilities] or a failure of the banking system to provide enough [bank liabilities] for use as currency. (Selgin 1988, p. 108)[22]

With the only circulating currency serving as base money, as is the case in modern fiat money systems, an increase in the demand for cash balances triggers an even larger contraction in the stock of deposit balances. The ability for banks to issue circulating banknotes as liabilities leveraged on top of the base money, rather than just deposits, would make it a matter of indifference whether the public holds its money balances as deposits or currency. The central bank would be absolved of the responsibility of adjusting to changes in the *composition* of money balances, and be tasked only with adjusting to changes in the velocity of spending – a task which, though still perhaps herculean, carries with it far less room for catastrophic error.

At that point, it is true, the tradeoff becomes more rigid. Once shifts in the form of money demand do not initiate changes in the economy's liquidity structure, once such changes are initiated only by shifts in demand for liquidity, strict automaticity of the money supply must be traded off against automaticity of the distribution of liquidity. But for now, our current system lies well within the institutional possibilities frontier.

VI. Conclusion

The significance of relative price distortions arising from monetary expansions has been mostly ignored since Friedman's thought experiment in which the expansion was distributed directly to consumers via helicopter drop. What began as a move for analytical convenience was solidified due to lack of evidence that such distortions were quantitatively relevant, even if they necessarily

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[22] I have replaced some equivalent terms for the sake of terminological consistency with the rest of this paper.

existed in principle. Even if helicopter drops weren't a good description of the mechanics of monetary expansion, relative price neutrality seemed to be a passable description of the effects.

This paper has subsumed that result into a wider framework in which monetary expansions are approximately neutral only to the extent that they are distributed through a banking system. Significant allocative inefficiency does not in fact arise from expansion through the banking system, but is likely to do so in any expansion that bypasses the banking system.

The conventional wisdom on the relative price effects of monetary policy holds because monetary expansion has typically been conducted through traditional open-market operations. Even recent unconventional expansion through quantitative easing has mainly centered on the banking system. For this reason, any allocative distortion arising from the spread of new money through the economy has been minimal. Should monetary policy turn in an even more unconventional direction, however, we should expect a great deal of nonneutrality in the future, for reasons well understood since the beginning of monetary economics.

Appendix. Analytical Solution for a Stationary Equilibrium

Removing the time subscripts from the helicopter model allows us to solve the model for a stationary equilibrium, which in turn allows us to prove that for a given set of parameter values, the foregoing model is formally equivalent to one in which the quantity of real balances demanded, rather than the weight of real balances in the utility function, is parametric. The same is true, with some added complication, for the helicopter model.

Ex ante, $E[\Delta V_g]=0$ and $E[v_g]=0$, ensuring that – per (1) – $E[P_g]=P_g$ – i.e., there exists a stationary equilibrium. Relative prices for any two goods j and a are given by $P_j / P_a = \mu_j \rho_j / \mu_a \rho_a$, which when substituting the equilibrium value of (3), becomes

$$\text{(A1)} \quad \frac{P_j}{P_a} = \sqrt{\frac{\mu_a}{\mu_j}}$$

We can introduce a nominal anchor by defining the money supply as identical to the sum of money balances held by agents and the store.

$$\text{(A2)} \quad M = b_s^* + \frac{N}{n} \sum_g b_g^* P_g$$

where b^*_g is the period-beginning real balances of a representative agent of type g, n is the number of agent types, and b^*_s is the store's nominal cash demand, which in equilibrium will equal its actual holdings at the beginning of the period.

Per (6), $b^*_s = wN$, which is equal to nominal GDP, accounting on the wage side rather than the goods side. Rearranging and substituting in (11), we have

$$\text{(A3)} \quad Y = M - \frac{N}{n} \sum_g \frac{b_g P_g}{\beta_g}$$

We therefore define GDP on the goods side as

$$\text{(A4)} \quad Y = \sum_g D_g P_g$$

Because production equals demand in equilibrium, $D_g = \mu_g N \rho_g$. Setting (A3) equal to (A4), we have

$$\text{(A5)} \quad \frac{M}{N} - \frac{1}{n} \sum_g \frac{b_g P_g}{\beta_g} = \frac{\sum_g \mu_g P_g^2}{\sum_g P_g}$$

Finally, per (11), (12), and (A1), and noting that $d_g = D_g/(N/n)$, we have

$$\text{(A6)} \quad b_g = \frac{n \beta_g \mu_g P_g}{(1 - \beta_g) \sum_h P_h}$$

Substituting (A6) and (A1), it is a matter of tedious algebra to show that

$$\text{(A7)} \quad P_g = \frac{M}{N} \frac{\sum_h \frac{1}{\sqrt{\mu_h}}}{\sqrt{\mu_g} \left(n + \sum_h \frac{1}{1 - \beta_h} \right)}$$

Substituting back into (A6), equilibrium real balances will therefore be

$$\text{(A8)} \quad b_g = \frac{\beta_g}{1 - \beta_g} \times \frac{n \sqrt{\mu_g}}{\sum_h \frac{1}{\sqrt{\mu_h}}}$$

It follows from (A8) that, provided the productivity parameters do not change, there will be a unique value of b_g for any β_g, and vice versa. One, therefore – though not both – may be regarded as parametric. Because b_g is highly nonlinear in β_g in the range of $\beta_g \in (0,1)$, and because we are more interested in linear perturbations of the demand for real balances than of parameters in the utility function, we shock b_g rather than β_g, and calculate the appropriate β_g on the basis of a parametric b_g.

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